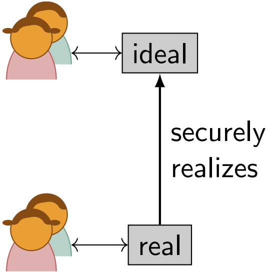
Formalizing Constructive Cryptography using CryptHOL

Andreas Lochbihler S. Reza Sefidgar David A. Basin Ueli Maurer

Digital Asset ETH zürich

Simulation-based Cryptography



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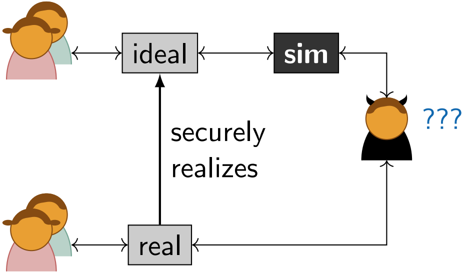

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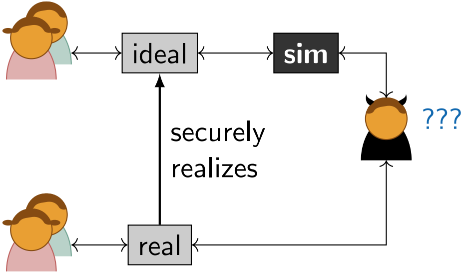

compositionality

Universal Composability

BPW

Constructive Cryptography

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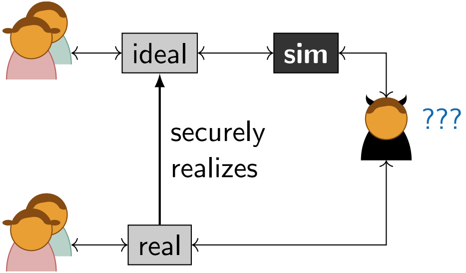

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Mechanic checks for cryptographic proofs to overcome the crisis of rigour

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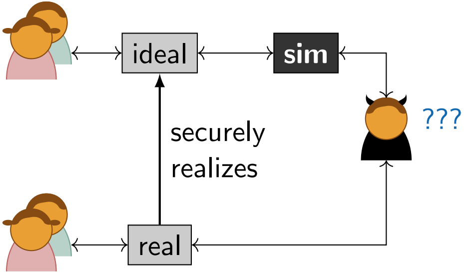

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CertiCrypt

CryptoVerif

EasyCrypt

FCF

CryptHOL

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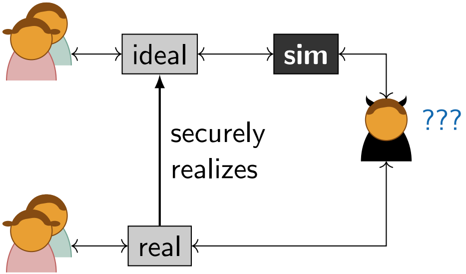

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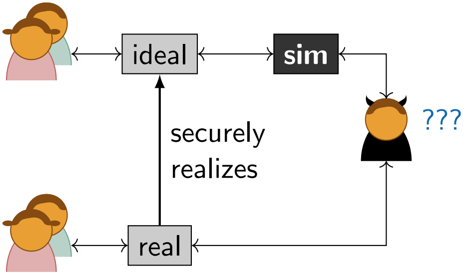

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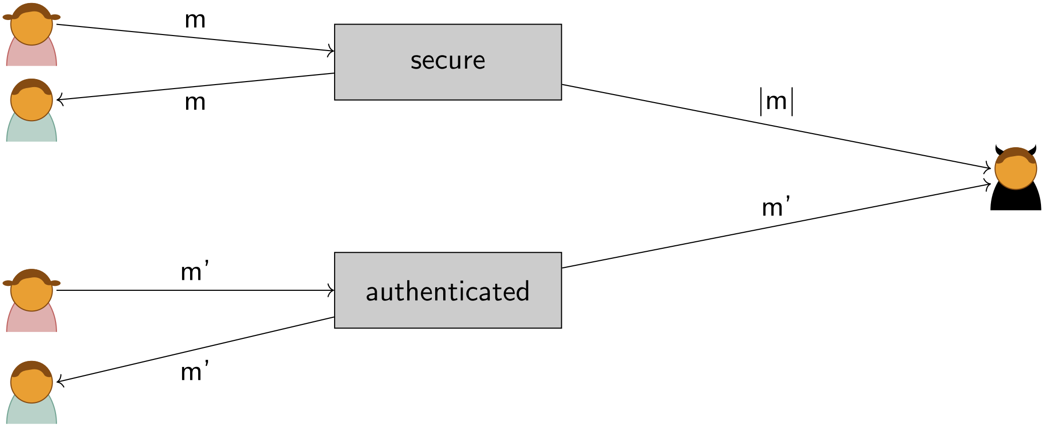
In this talk:

- CC formalization in Isabelle/HOL (information-theoretic security)
- proof of compositionality
- application to a case study (insecure channel $\rightsquigarrow$ secure channel)

CryptHOL

Channels in Constructive Cryptography



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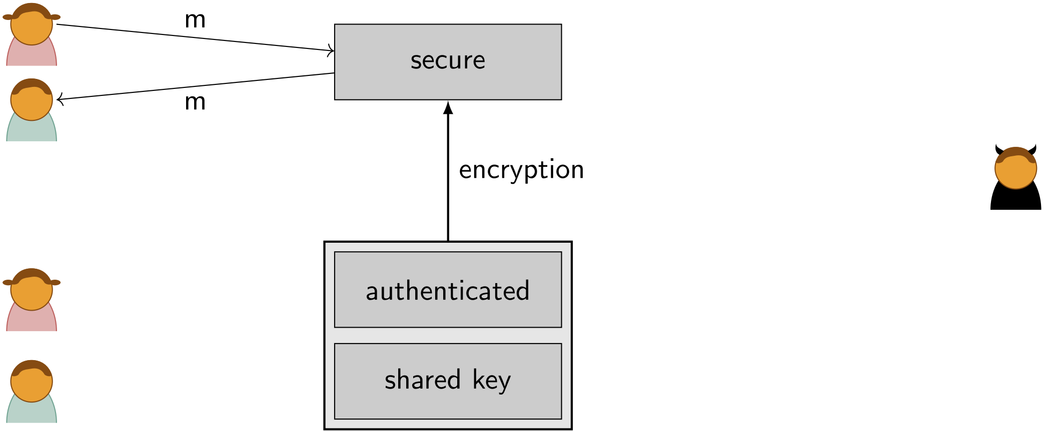

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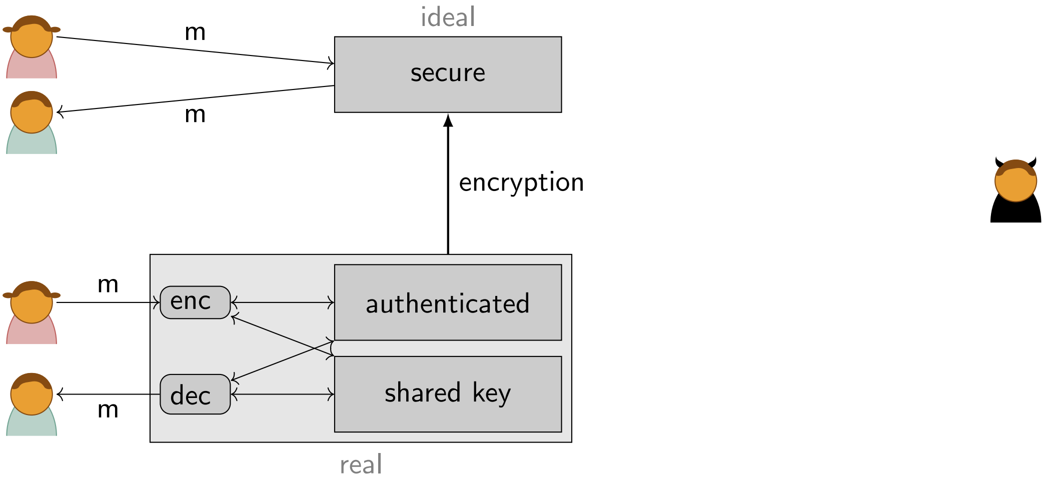

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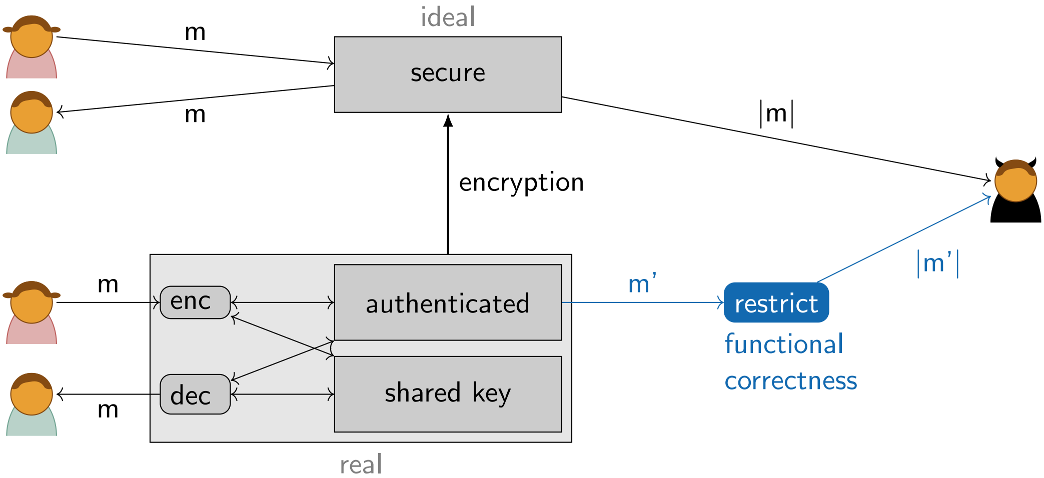

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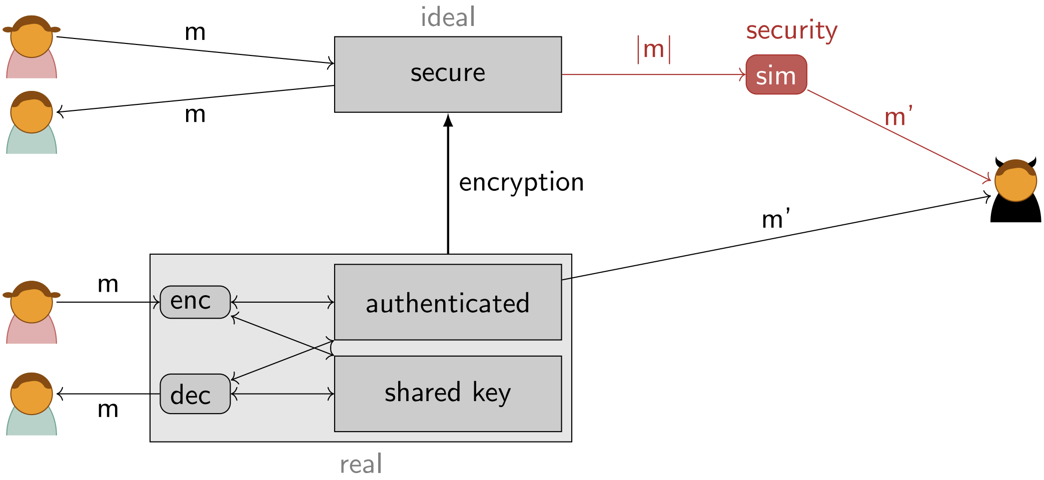

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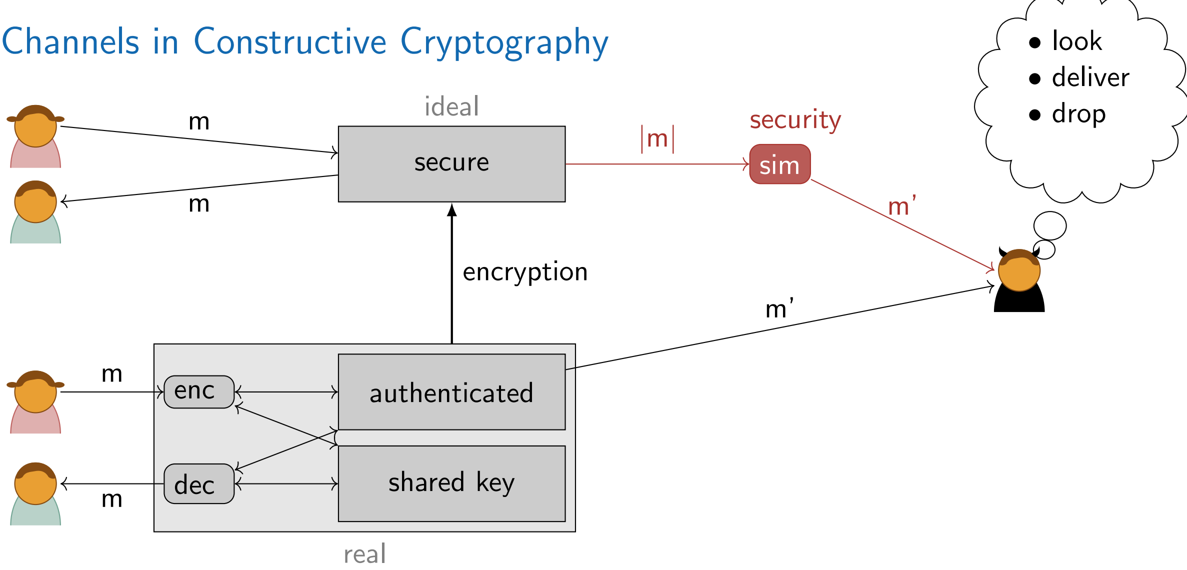

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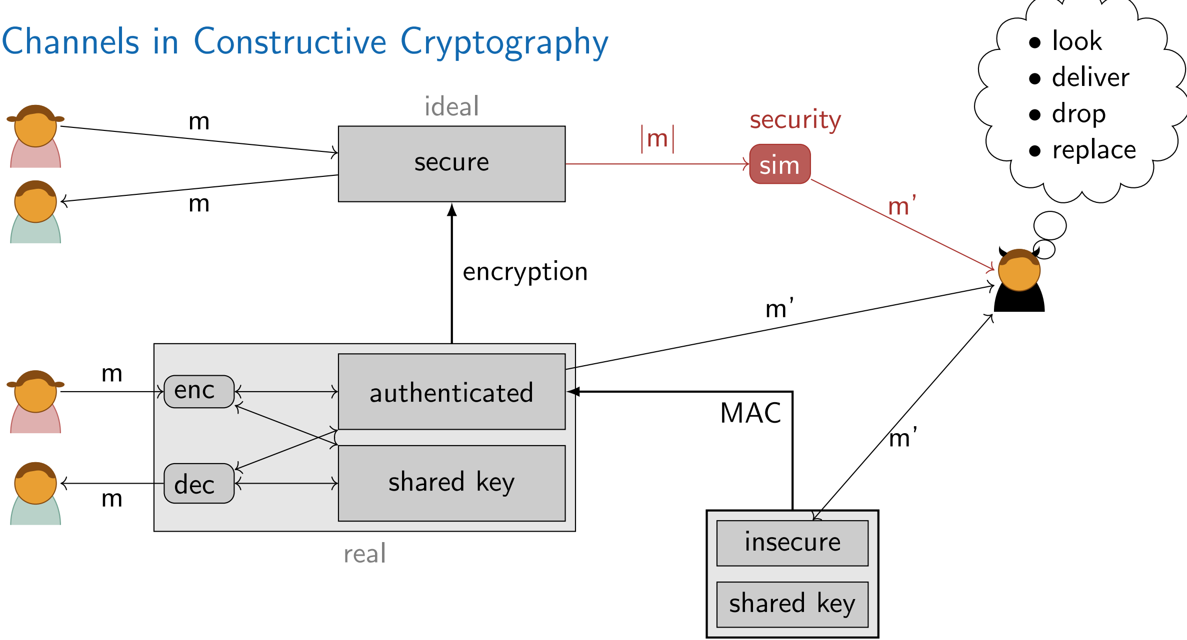
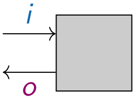


Formalizing Resources



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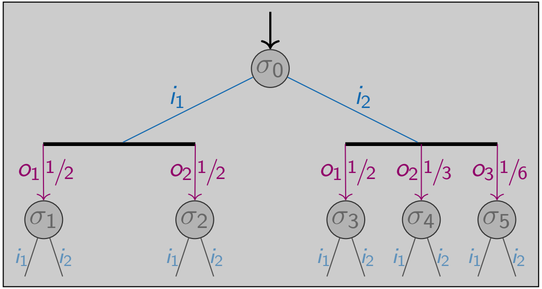
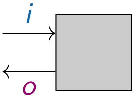


1. Probabilistic transition system (d, σ_0)

$$
\begin{aligned}
d &: \Sigma \to I \to \mathbb{D}(O \times \Sigma) \\
\sigma_0 &: \Sigma
\end{aligned}
$$

(= CryptHOL oracle)

Formalizing Resources



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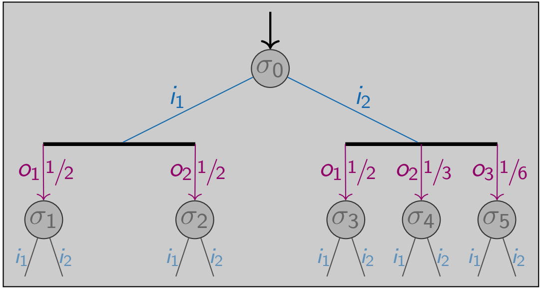

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---

2. Abstract over the concrete state

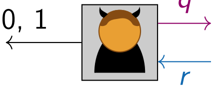
$$\exists \Sigma.\ (\Sigma \to I \to \mathbb{D}(O \times \Sigma)) \times \Sigma$$

codatatype $\mathbb{R}(I, O) =$
$\quad$ $\mathit{Resource}\ (I \to \mathbb{D}(O \times \mathbb{R}(I, O)))$

Benefits

- Identifies bisimilar resources
- Can exploit **corecursive structure** (unwinding) in definitions and proofs

Formalizing Distinguishers (≈ CryptHOL Adversary)



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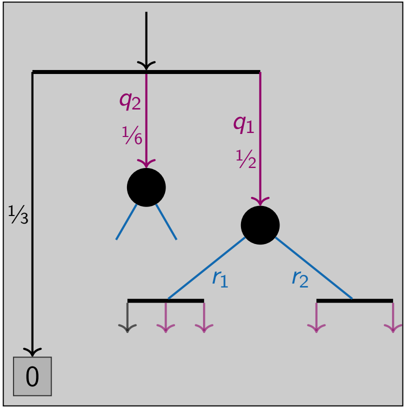

CryptHOL: Generative probabilistic value (GPV) + probabilistic termination

$$\texttt{codatatype}\ \mathbb{G}(A, Q, R) = Gpv\ (\mathbb{D}(A + (Q \times (R \to \mathbb{G}(A, Q, R)))))$$

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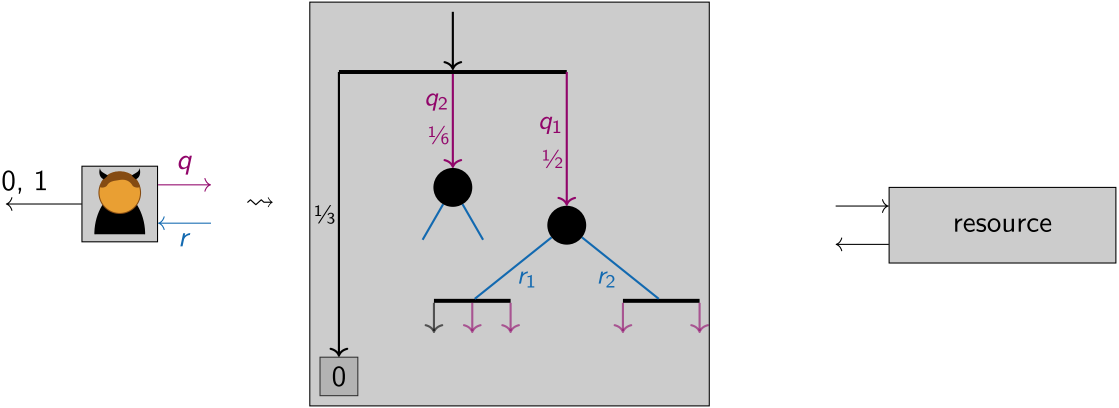

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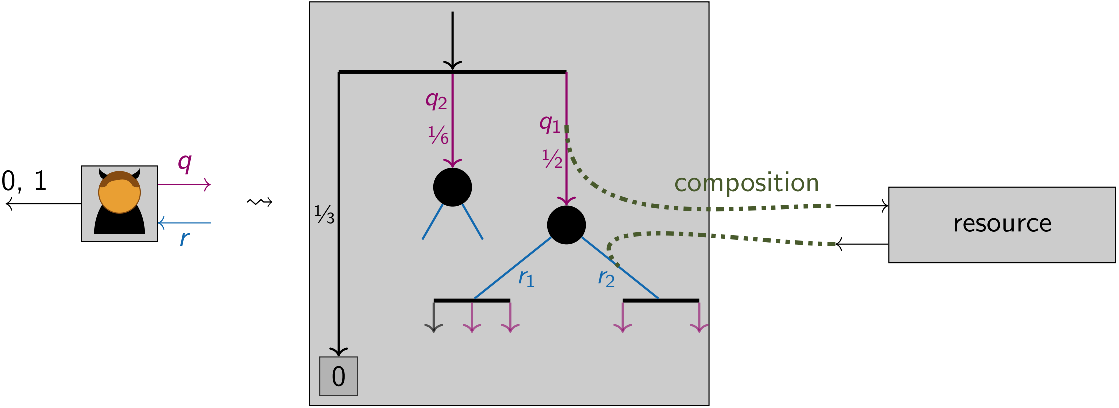

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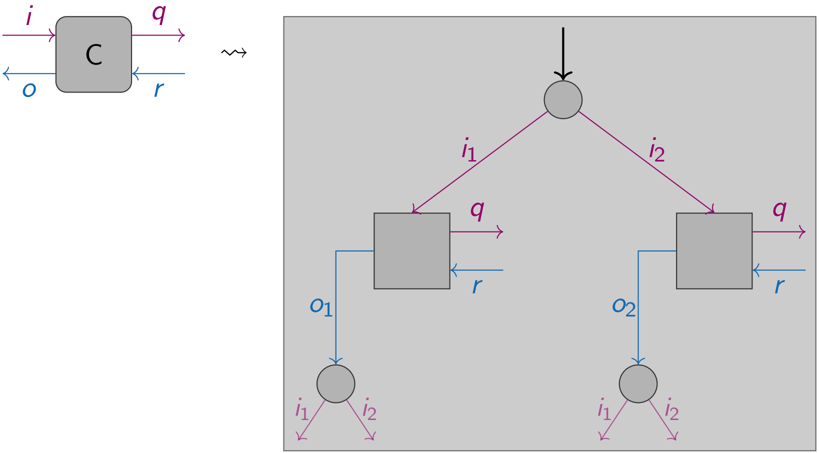
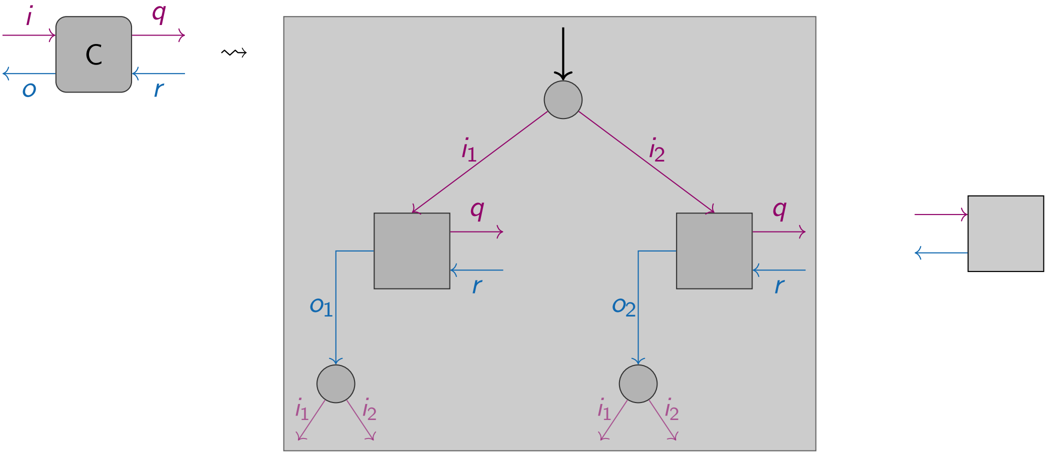
Formalizing Converters



$$\texttt{codatatype}\ \mathbb{C}(I, O, Q, R) = \mathit{Converter}\ (I \to \mathbb{G}(O \times \mathbb{C}(I, O, Q, R), Q, R))$$

Formalizing Converters



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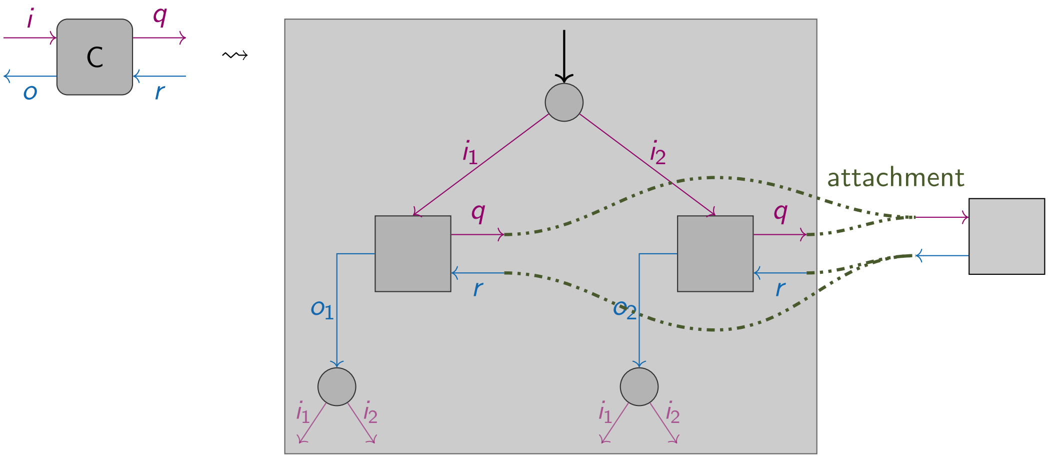

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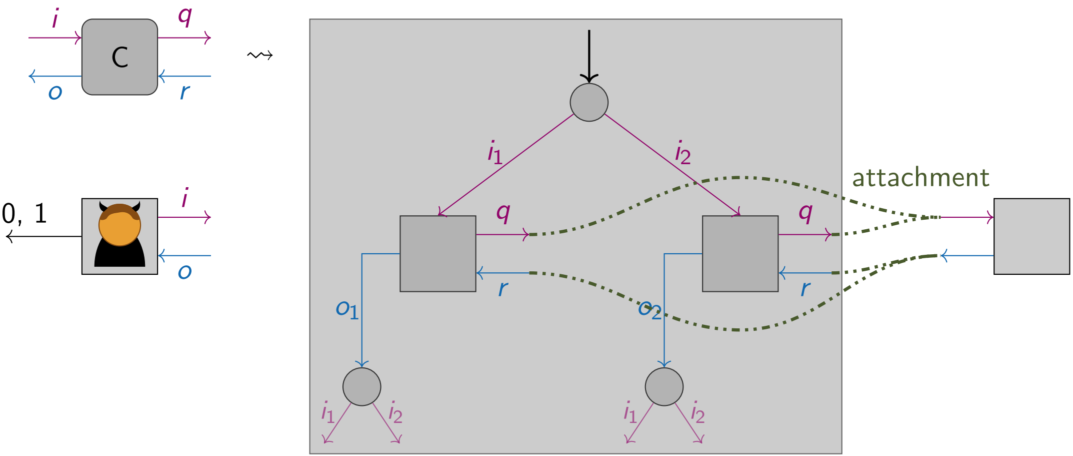

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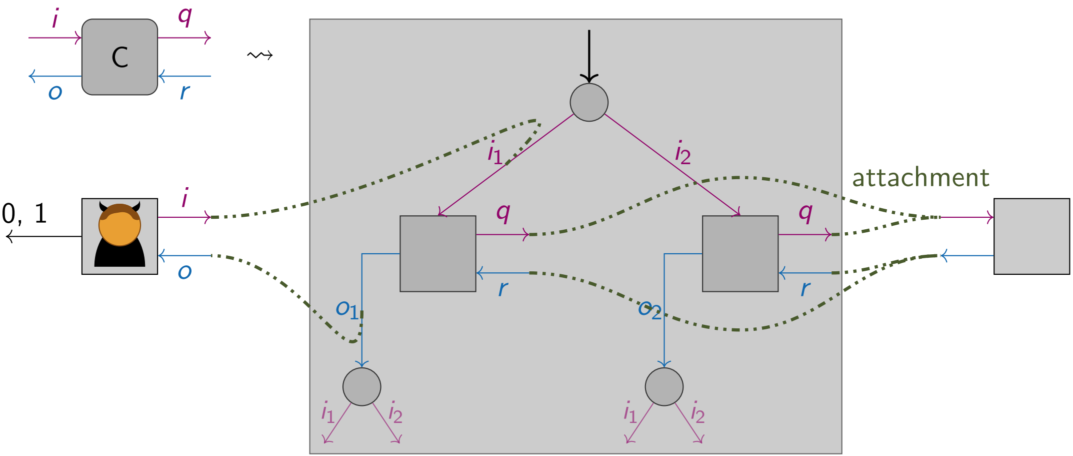

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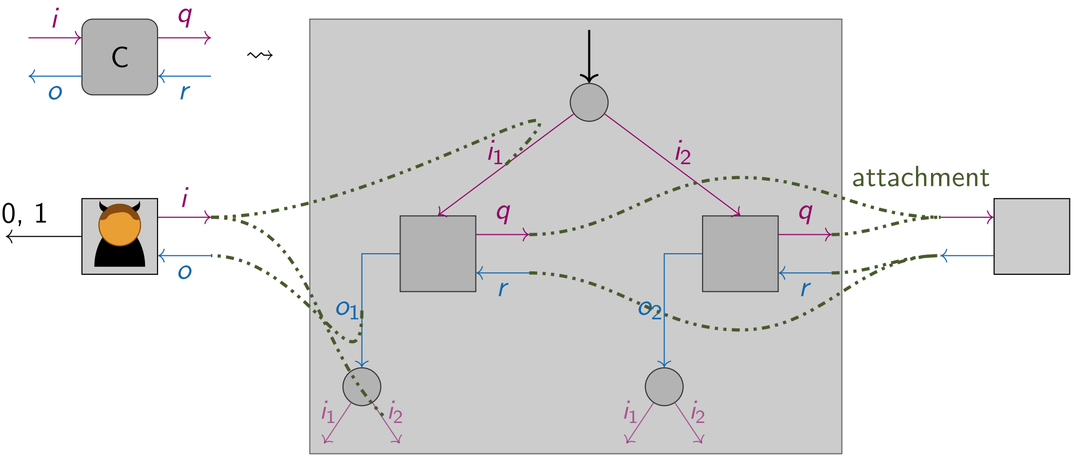

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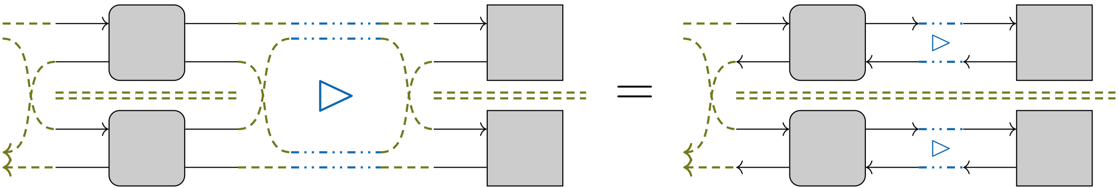
Algebraic Reasoning

```
lemma attach_parallel2:
  "(C1 |= C2) ▷ (R1 ∥ R2) = (C1 ▷ R1) ∥ (C2 ▷ R2)"
```



Algebraic Reasoning Becomes Simpler

Abstraction over state simplifies reasoning about composition

```
lemma attach_compose:
  "(C1 ⊙ C2) ▷ R = C1 ▷ (C2 ▷ R)"
```

Algebraic Reasoning Becomes Simpler

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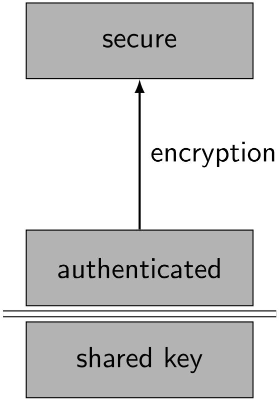
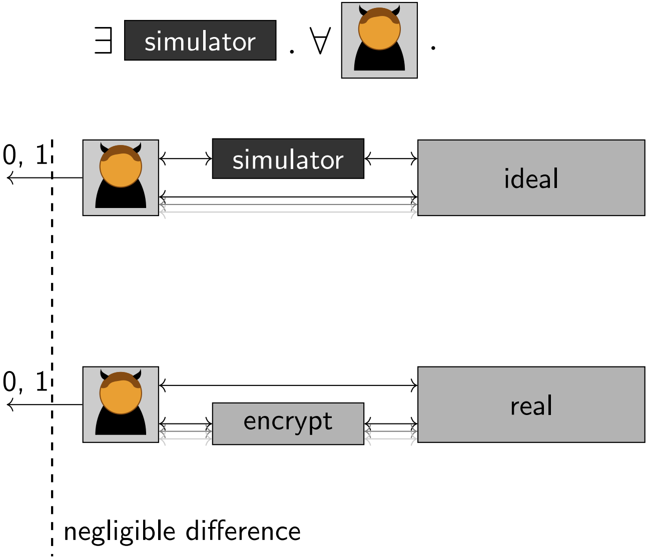
lemma attach_compose:
  "(C1 ⊙ C2) ▷ R = C1 ▷ (C2 ▷ R)"
```

In CryptHOL:

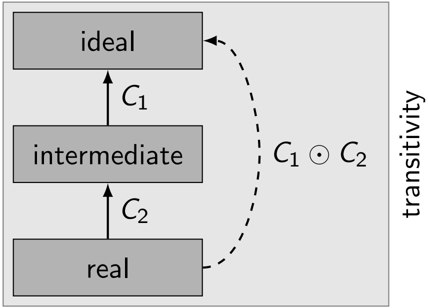
```
lemma exec_gpv_inline:
  "exec_gpv R (inline C2 C1 s') s =
   map_spmf (λ(x, s', s). ((x, s'), s)) (exec_gpv
     (λ(s', s) y. map_spmf (λ((x, s'), s). (x, s', s))
       (exec_gpv R (C2 s' y) s))
     C1 (s', s))"
```

Formalizing Secure Realization (asymptotic version)





Formalized Composition Theorems



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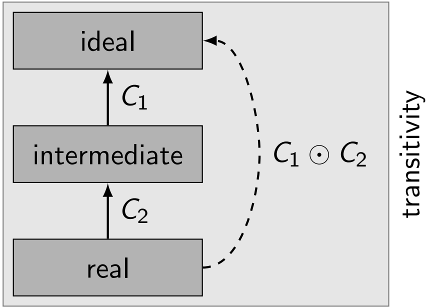

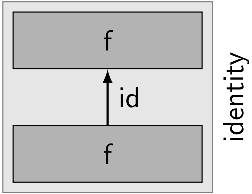

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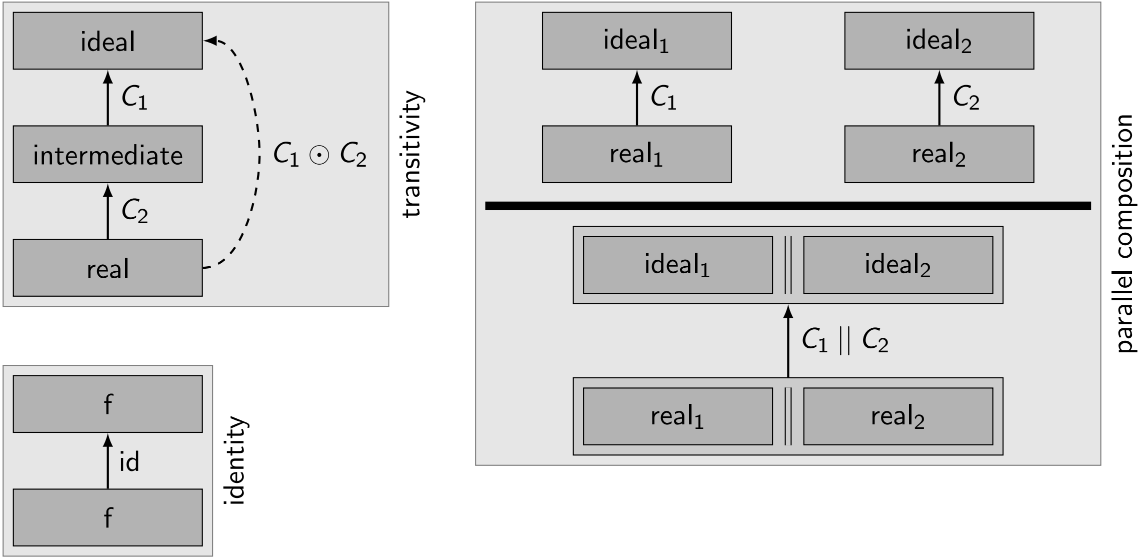
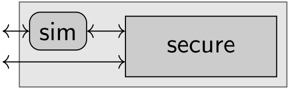


Example: One-time-pad Encryption over a Single-use Channel

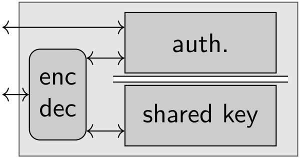
Interfaces

Resource	Users	Adversary
secure channel	submit / poll	**length**, deliver, drop
authenticated ch.	submit / poll	**look**, deliver, drop
shared key	get	—



Encrypt:

1. get key
2. XOR key with message
3. submit



Decrypt:

1. get key
2. poll message
3. XOR key with message

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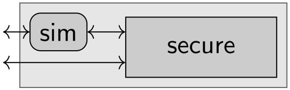

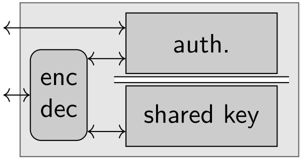

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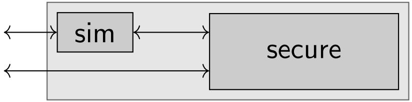
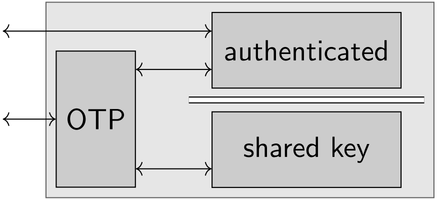
2. poll message
3. XOR key with message

Simulator:

authenticated	$\mapsto$	secure channel
look	$\mapsto$	length + sample bitstring
deliver	$\mapsto$	deliver
drop	$\mapsto$	drop

Proof Approach





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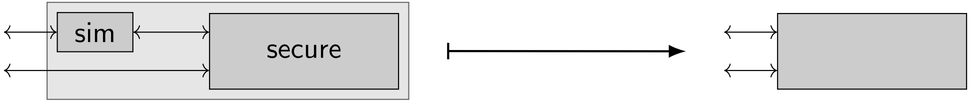

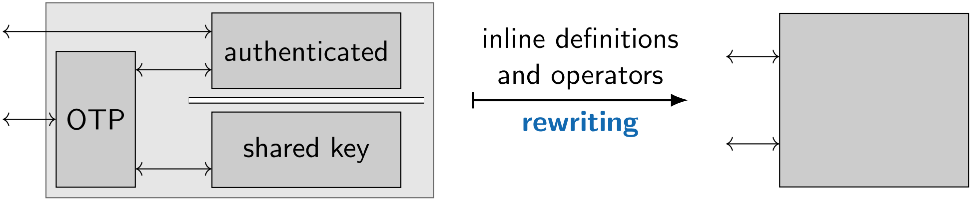

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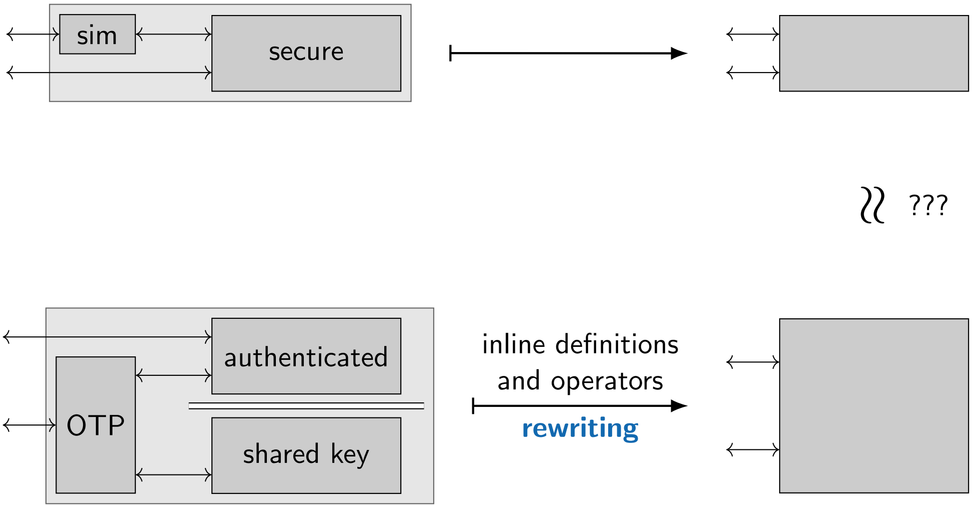

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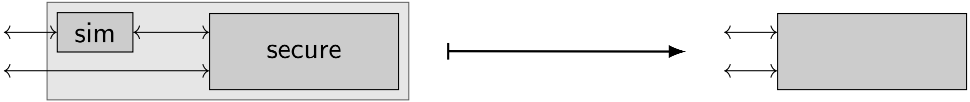
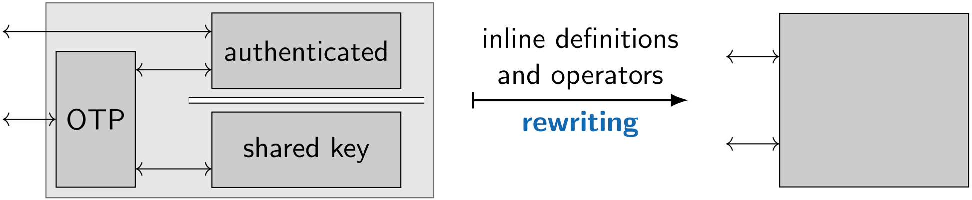


Attempt 1: Bisimulation
relation between states of the resources must be preserved by every interaction
$\rightsquigarrow$ local reasoning

$\wr$???



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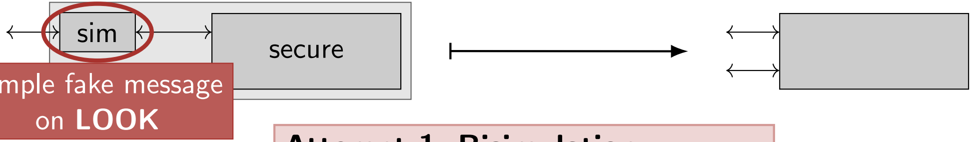

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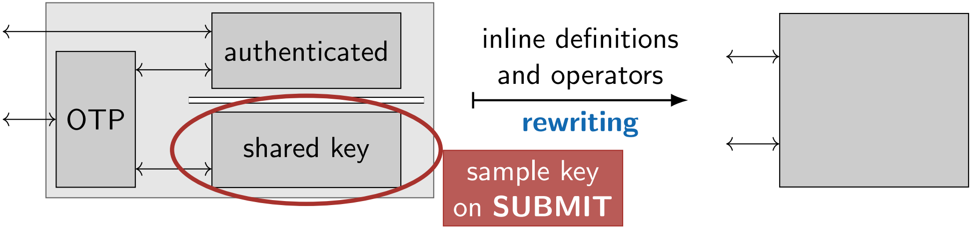
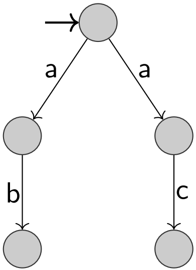
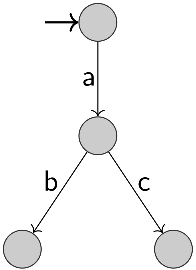
≀ ???



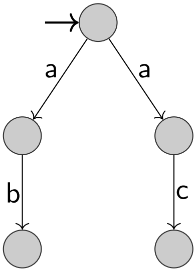
Why Bisimulation is too Strong





Why Bisimulation is too Strong

OTP



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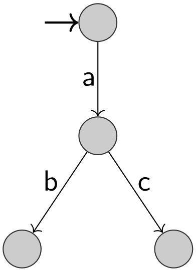

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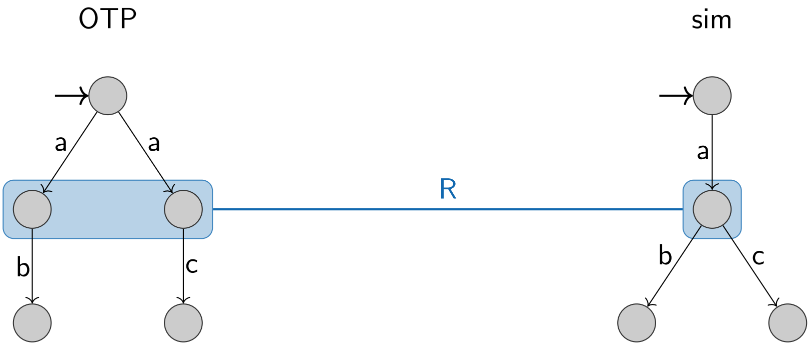

Attempt 2: Trace Equivalence

Random system [Maurer'02]: Family of conditional probability distributions

previous interactions		next input		conditional distribution over next output
$[I \times O]$	$\rightarrow$	I	$\rightarrow$	$\mathbb{D}(O)$

$$(\Sigma \rightarrow I \rightarrow \mathbb{D}(O \times \Sigma)) \times \quad \Sigma$$

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$$\Uparrow \textit{trace}$$

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$\Uparrow$ *trace*

recursive definition

$$(\Sigma \rightarrow I \rightarrow \mathbb{D}(O \times \Sigma)) \times \mathbb{D}(\Sigma)$$

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$\Uparrow$ *trace* recursive definition

$$(\Sigma \rightarrow I \rightarrow \mathbb{D}(O \times \Sigma)) \times \mathbb{D}(\Sigma)$$

Characterization theorem:
Two resources are trace equivalent
iff the distinguishing advantage is 0.

Attempt 2: Trace Equivalence

Random system [Maurer'02]: Family of conditional probability distributions

previous interactions		next input		conditional distribution over next output
$[I \times O]$	$\rightarrow$	I	$\rightarrow$	$\mathbb{D}(O)$

$\Uparrow$ *trace* recursive definition

$$(\Sigma \rightarrow I \rightarrow \mathbb{D}(O \times \Sigma)) \times \mathbb{D}(\Sigma)$$

Characterization theorem:
Two resources are trace equivalent
iff the distinguishing advantage is 0.

Sound and complete **unwinding proof rule**
Local, simulation-like proof principle
for trace equivalence

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Suffices to complete the proofs

definition

$$\| $$

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	CryptHOL	**FCF**	**EasyCrypt**
Underlying technology	Isabelle/HOL	Coq	OCaml
Definitional approach	⊕	⊕	⊕
Expressive codatatypes	⊕	0	⊖
Library	⊕	0	growing
Dependent types	⊖	⊕	⊖

Take aways

1. Coalgebraic modelling $\rightsquigarrow$ mechanized algebraic reasoning
2. Trace equivalence is the right equivalence notion
3. Unwinding proof rule for trace equivalence
4. Formalization suitable for abstract (composition) and concrete (OTP, MAC) reasoning

`www.isa-afp.org/entries/Constructive_Cryptography.html`

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More in the paper

- Dependent type system
  for resources and converters
- Formalization of wiring

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Future work

- Further applications
- Computational security

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